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13. ABSTRACT (Maximum 200 words)

The subject of the research is detection and estimation employing an array of sensors. Of particular concern is efficient and numerically reliable computational strategies for implementing prevalent detection/estimation procedures. A number of important array processing problems lead to a differencing of matrix outer products. This leads to potential ill-conditioning when implemented explicitly. The avoidance of outer products, at the expense of extra operations, has long been a crusade of sorts in the numerical analysis community (Golub). One can do without outer products by means of orthogonal, or for complex data unitary, transforms in the usual case where a sum of outer products arise. (Typical transforms that have been found to be particularly useful are Givens, Jacobi, and Householder transforms). The idea is to transform the data into sparse form while preserving its pertinent statistics (usually the sample convariance matrix). This research concerns generalizing this "trick" to the case of a difference of outer products by means of hyperbolic, rather than orthogonal transforms.

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The task of removing data (downdating) arises in sensor arrays if: (i) recursive block processing is employed for a nonstationary environment or (ii) robust estimation is employed to remove contamination from outliers [Haykin].

Our contribution to this problem is the development of a stabilized hyperbolic transformation which solves this problem [1],[7], [14], [18]. We have proven that our algorithm is indeed stable [1], and it gives more accurate results than all competing methods in simulations [7],[14],[18]. The central ideas behind the algorithm are as follows. First the problem statement, in mathematical terms is:

> Given: An n by l matrix X_1, an n by p matrix X_2 and a lower triangular (Cholesky) matrix L, with $LL^t = X_1X_1^t$ we seek the new Cholesky matrix $\tilde{L}$ formed from downdating X_2, that is we seek $\tilde{L}$ such that:
>
> $$\tilde{L}\tilde{L}^t = LL^t - X_2X_2^t. \tag{1}$$

(Once we have obtained the "sparse" covariance-invariant matrix $\tilde{L}$ the least squares solution is easy to compute.) Several computational solutions to (1) already exist which avoid outer products, as outlined in [1]. The most computationally efficient of these is the hyperbolic Householder transform [Rader], developed by the author with C. Rader at Lincoln Laboratory. Unfortunately, although less ill-conditioned than outer product schemes, it is apparently not backwards stable. We modify it to make it stable in a fashion outlined below.

Suppose we were given $\tilde{L}$ and X_2, and we sought L. Then we would have: $L = [\tilde{L}|X_2]Q$ for some orthogonal matrix Q, readily obtainable by standard techniques. (In reality $\tilde{L}$ is unknown and L is known.) The key trick is this: form Q and $\tilde{L}$ "on the fly", exploiting the fact that (i) Q is orthogonal, (ii) $\tilde{L}$ is lower triangular and (iii) the result of the concomitant matrix product is the known matrix L. This trick yields a tractable, albeit nonlinear, recursive procedure for obtaining $\tilde{L}$ and Q jointly. The recursions involve hyperbolic transforms to create Q, but once Q is formed only orthogonal transforms (which are optimally stable) are applied to the data.

Our new stabilized algorithm is the "best" choice (measured in terms of computational efficiency and numerical accuracy) except in certain cases outlined in the conclusion of [7].

1.2 Hyperbolic singular value decomposition

Consider the following problem:

> Given an n by l matrix X_1, and an n by p matrix X_2 find the eigenvectors of the matrix
>
> $$\Delta R = X_1X_1^t - X_2X_2^t. \tag{2}$$

This problem arises in various array processing tasks:

- Bearing estimation in colored noise using covariance differencing [Paulraj],

- recursive block processing whenever eigenanalysis is employed in each block (e.g. high resolution bearing estimation) [Haykin], and

- Array calibration with mismatched sensors. [Kaveh] Sensor mismatch is ubiquitous, arising from differing antenna patterns, variations in the IF and I/Q stages, etc. Mismatch, if ignored, leads to serious performance degradation. Therefore this is an application particularly deserving of widespread interest.

Our contribution to this problem is the introduction of the hyperbolic singular value decomposition (HSVD) [2],[8], [12], [13], [15], [16], [17], [19]. The HSVD is a new canonic matrix decomposition which we define by

$$A = UDV, \quad \text{where } U \text{ is unitary, } D \text{ is diagonal, and} \quad V\Sigma V^t = \hat{\Sigma}, \tag{3}$$

where $\Sigma, \hat{\Sigma}$ are signature matrices (i.e., diagonal with entries ± 1), and A is the initial given matrix to be decomposed. Note that in V we again have a hyperbolic transformation. The well known conventional SVD yields the eigenstructure of $X_1 X_1^t$ directly from X_1 [Golub]. Likewise the HSVD yields the eigenstructure of ΔR directly (without outer products) from the matrix $A = [X_1 | X_2]$. More specifically, one can readily see that with $\Sigma = \mathrm{diag}(I, -I)$ the U matrix in the HSVD contains the eigenvectors of ΔR, and $D\hat{\Sigma}D^t$ contains its eigenvalues. We establish the canonicity of (3) in [19], [13]. How does one compute the HSVD? We have constructed two HSVD algorithms, one serial (based on Golub-Kahan SVD [Golub]) and one parallel (based on Hestene's SVD). Simulations show that, like the SVD, these HSVD algorithms are numerically well behaved [15]. We emphasize that the HSVD is the only currently available algorithm which avoids outer products while rendering a solution to (2) for arbitrary A.

1.3 Dissertations

The stabilized Hyperbolic Householder was the topic of an MS Thesis by S. So, "Complex Stabilized Downdating in Adaptive Beamforming", Dec. 1989.

The HSVD was the (partial) topic of a PhD Thesis by R. Onn, Dept. EE, Cornell University, Jan. 1992, "Linear Algebra in Sensor Array Processing".

The grant provided some (minimal) stipend for these students.

1.4 Research Impact

A number of researchers have expanded on our work. Here is a partial listing.

The stabilized Hyperbolic Householder work was extended by Lektomeki at the last ICASSP in Toronto (April 1991) in the context of sonar detection.

The hyperbolic Householder has also been applied to data adaptive windowing at ICASSP 1991 by Hseih, and to subspace updating by Bischoff in a Jan. 1992 IEEE SP Transactions article. The Hyberbolic Householder has also been applied by G. Cybenko of U. Illinois in

List of Publications supported by AFOSR-89-0267

Most of these publications (denoted by *) address hyperbolic transforms. Those that do not are initial explorations into enhanced convergence, which will be detailed further in the final report for "Enhanced Convergence Adaptive Nulling", AFOSR-91-0149.

Refereed Journal Articles

1. * "Stability analysis of a Householder based algorithm for downdating the Cholesky factor", A. Bojanczyk and A. Steinhardt, *SIAM Journal of Sci. Computing*, Sept. 1991.

2. * "Hyperbolic Singular Value Decomposition and Applications", R. Onn, A. Steinhardt, and A. Bojanczyk, *IEEE Trans. on Acoustics, Speech, and Signal Processing*, July, 1991.

3. "On the PDF of adaptive beamforming weights", A. Steinhardt, *IEEE Trans. on Acoustics, Speech, and Signal Processing*, May, 1991, pp.1232-1235.

4. "'On the peaks of causal signals with a given average delay", J. Makhoul and A. Steinhardt, *IEEE Trans. on Acoustics, Speech, and Signal Processing*, April, 1991 pp.620-625.

5. "On matching Correlation sequences by parametric spectral models", J. Makhoul and A. Steinhardt, *IEEE Trans. on Acoustics, Speech, and Signal Processing*, Jan. 1991, pp.214-216.

6. * "Adaptive Beamforming", A. Steinhardt and B. Van Veen, *Int. Journal Adaptive Control and Signal Processing*, Sept. 1989, pp.253-281.

7. * "'Stabilized Hyperbolic Householder Transformations", A. Bojanczyk and A. Steinhardt, *IEEE Trans. on Acoustics, Speech, and Signal Processing*, Aug. 1989, pp.1286-1288.

Invited Presentations

8. * "The Hyperbolic Singular Value Decomposition and Applications", A. Bojanczyk, R. Onn and A. Steinhardt, *SVD and Signal Processing II, Algorithms, Analysis and Applications*, edited by R.J. Vaccaro, Elsevier, 1991, pp.132-148. (*Proc. IEEE SP SVD Wrkshp., RI, June 1990.*)

Books

9. *Adaptive Radar detection and estimation*, S. Haykin, A. Steinhardt, Editors, Wiley, to appear 1992. Also author of chapter 2, "Adaptive multisensor detection and estimation",to appear, March 1992.

Conference Presentations

10. "Where's the Peak?", J.Makhoul and A. Steinhardt, *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, Toronto, Canada, May, 1991, pp.3361-3364.

11. "The peak of a causal signal with a given average delay", J.Makhoul and A. Steinhardt, *Proc. 5th ASSP Workshop on Spectrum Estimation,* Rochester, NY, Oct. 1990, pp.246-249.

12. * "The hyperbolic SVD and applications", R. Onn, A. Steinhardt, and A. Bojanczyk, *Proc. 5th ASSP Workshop on Spectrum Estimation,* Rochester, NY, Oct. 1990, pp.285-288.

13. * "Hyperbolic SVD, a new canonic decomposition", R. Onn, A. Steinhardt, and A.Bojanczyk, SIAM Conf. on Linear Alg. and Appl., San Diego, CA, July, 1990.

14. * "Stable complex downdating in adaptive beamforming", S. So and A. Steinhardt, *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, Alb., NM, May 1990, pp.2659-2662.

15. * "The hyperbolic SVD and applications", R. Onn, A. Steinhardt, and A. Bojanczyk , *Proc. 32nd Midwest Symposium on Circuits and Systems,* U. of Ill., Aug. 1989 pp.575-577.

16. * "A linear array for covariance differencing via hyperbolic SVD", A. Bojanczyk and A. Steinhardt, *Proc. SPIE Conf. on Advances in Signal Processing,* San Diego, Aug. 1989 pp.103-107.

17. * "A systolic array for hyperbolic SVD based covariance differencing", A. Bojanczyk and A. Steinhardt, *Proc. 3rd Int. Conf. on Systolic Arrays,* Kilnrey, Ireland, May 1989, pp.247-254.

18. * "Stabilized hyperbolic Householder transforms", A. Bojanczyk and A. Steinhardt, *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, Glascow, Scotland, May 1989, pp. 1278-1281.

In Review and To Appear

19. * "Hyperbolic singular value decomposition", R. Onn, A. Bojanczyk. and A. Steinhardt, *J. Lin. Al. Appl.*, in second stage of review cycle.

recent work to appear in Int. J. Linear Algebra and Applications in the context of parallel computation of low displacement rank.

Chen, Kailath, and Luk have used HSVD concepts in singular displacement rank theory.

Two new textbooks discuss our Hyperbolic Householder device; (i) S. Haykin,"Adaptive Filter Theory", Wiley, 1991, and (ii) S. Farina, "Radar Signal Processing", Artech House, 1992.

We mention for completeness independent work by DeMoor (U. Ill.) which generalizes the HSVD, by generalizing the form of $\hat{\Sigma}$. (De Moor has since incorporated our work in his presentation.)

[Golub] G.Golub and C. Van Loan, *Matrix Computations*, John Hopkins Press, 1983.

[Monzingo] R.Monzingo and T.Miller, *Introduction to Adaptive Arrays* Wiley,New York,1980.

[Haykin] S. Haykin,*Adaptive Filter Theory*, Prentice-Hall, 1986.

[Rader] C.Rader and A.Steinhardt, "Hyperbolic Householder Transformations", IEEE Trans. Acoust., Speech, Signal Proc., (*IEEE ASSP*), Dec. 1986, also SIAM J. on Matrix Anal. and Appl., April, 1988.

[Paulraj] A.Paulraj and T.Kailath,"Eigenstructure methods for direction of arrival estimation in the presence of unknown noise fields", *IEEE ASSP*, Feb. 1986.

[Kaveh] J.Pierce, M.Jacobsen, and M.Kaveh, "A Laboratory testbed for sensor array processing", 32nd midwest symp. on circuits & syst., Champ.,Ill., 1989 M.S. Thesis, Cornell University, in preparation.